Roll No.__________ Reg. No __________ Date__________ Sign__________

GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

QUESTION PAPER: EXTERNAL EXAMINATIONS

M Sc Mathematics | Part 1st | 1st . Annual 2015
Course Code: MATH-C11 | Course Title: Real Analysis

OBJECTIVE PART | Time Allowed: 30 Minutes | Marks: 20

Note: This question No.1 is compulsory and its all parts carry equal marks. Please attempt the answers on same paper and return it to center superintendent with in the time allowed.

Q# 01:(a) Answer of the following short questions.

(i) Let S be a non-empty subset of R that is bounded below. Prove that $\inf S = -\sup\{-s : s \in S\}$.

(ii) Discuss the convergence of the following sequence, where b satisfy $b > 1$, $(\frac{b^n}{n!})$.

(iii) Prove that $\lim_{x\to 0} \cos(\frac{1}{x})$ does not exist but that $\lim_{x\to 0} x\cos(\frac{1}{x}) = 0$.

(iv) Evaluate $\lim(\frac{nx}{1+n^2x^2}) = 0, \forall x \in R$.

(v) Discuss the convergence or divergence of the series with the nth term $n!e^{-n}$.

Q # 01: (b) Match the column A with the column B and select the correct answer from "B" and write it in column C.

Column A	Column B	Column C
If $a,b \in R$, $\max\{a,b\}$	divergent	
If $a,b \in R$, $\min\{a,b\}$	convergent	
$\sup\{1-\frac{1}{n} : n \in N\}$	$2\int_0^a f$	
$\lim\left((2n)^{\frac{1}{n}}\right)$	0	
$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$	$\frac{1}{2}(a+b+\lvert a-b\rvert)$	
$\sum_{n=1}^{\infty} \frac{1}{n!}$	$\frac{1}{2}(a+b-\lvert a-b\rvert)$	
f is even, $\int_{-a}^{a} f =$	1	
f is odd, $\int_{-a}^{a} f =$	1	
$\lim\left(\frac{n^2-1}{2n^2+3}\right) =$	-1	
$S := \{\frac{1}{n} - \frac{1}{m} : n, m \in N\}, \inf S$	$\frac{1}{2}$	

Roll No..........

GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

QUESTION PAPER: EXTERNAL EXAMINATIONS

M Sc Mathematics | Part 1st | 1st Annual 2015

Course Code: MATH-C11 | Course Title: Real Analysis

Time Allowed: 03:00 Hours | Maximum Marks: 100 | Pass Marks: 40%

SUBJECTIVE PART | Time: 02:30 Hours | Marks: 80

Section-I

Note: *Attempt any two questions. All questions carry equal marks.*

Q# 02:

(a) Let $S \subseteq R$ and suppose that $s^* := \sup S$ belongs to S. If $u \notin S$, show that $\sup(S \cup \{u\}) = \sup\{s^*, u\}$.

(b) Let $\varepsilon > 0$ and $\delta > 0$, and $a \in R$. Show that $V_\varepsilon(a) \cap V_\delta(a)$ and $V_\varepsilon(a) \cup V_\delta(a)$ are γ-neighborhoods of a for appropriate values of γ.

Q # 03:

(a) Every contractive sequence is a Cauchy sequence, and therefore is convergent.

(b) The series $\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}$ is convergent.

Q # 04:

(a) $A \subseteq R$, let $f : A \to R$ and let $c \in R$ be a cluster point of A. If $\lim_{x \to a} f$ exists, and if $|f|$ denotes the function defined for $x \in A$ by $|f|(x) := |f(x)|$, prove that $\lim_{x \to c} |f| = \left|\lim_{x \to a} f\right|$.

(a) $I := [a,b]$ be a closed bounded interval and let $f : I \to R$ be continuous on I. Then f is bounded on I.

Q # 05:

(a) Use the Mean value Theorem to prove that $(x-1)/x < \ln x < x - 1$ for $x > 1$.

(b) Let I be an interval and let $f : I \to R$ be strictly monotone on I. Let $J := f(I)$ and let $g : J \to R$ be the function inverse to f. If f is differentiable on I and $f'(x) \neq 0$, for $x \in I$, then g is differentiable on J and $g' = \frac{1}{f' \circ g}$.

Section-II Attempt any Two questions.

Q # 06:

(a) If $u = f(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$, then show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2.$$

(b) Find the point of the curve $x^2 - xy + y^2 - z^2 = 1, x^2 + y^2 = 1$ nearest to the origin $(0,0,0)$.

Q # 07:

(a) If $f : [a,b] \to R$ is continuous on $[a,b]$, then $f \in R[a,b]$.

P-T-O

(b) Let $B(x) := -\frac{1}{2}x^2$ for $x < 0$ and $B(x) := \frac{1}{2}x^2$ for $x \geq 0$. Show that $\int_a^b |x|\,dx = B(b) - B(a)$.

Q # 08:.

(a) Let $(f_n).(g_n)$ be a sequences of bounded functions on A that converges uniformly on A to $f, g,$ respectively. Show that $(f_n g_n)$ converges uniformly on A to fg.

(b) The limit of a power series is continuous on the interval of convergence. A power series can be integrated term-by-term over any closed and bounded interval contained in the interval of convergence.

Q # 09:

(a) If f is continuous for $-\infty < x < \infty$ and f is odd then $\int_{-\infty}^{\infty} f(x)dx = 0.$

(b) Use the known result that $\int_0^{\infty} e^{-x^2} dx = \frac{1}{2}\sqrt{\pi}$, to prove that $\int_0^{\infty} e^{-xt^2} dt = \frac{1}{2}\sqrt{\frac{\pi}{x}}, x > 0.$

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GOVERNMENT COLLEGE UNIVERSITY,FAISALABAD

QUESTION PAPER EXTERNAL EXAMINATION

M.Sc Mathematics Part 1st 1st Annual 2015

Course Code MTH-C12 Course Title: ALGEBRA Pass Marks 40%

OBJECTIVE PART	Time ALLOWED: 30 Minute	Marks 20

Note:This question No 1 is compulsory and its all parts carry equal marks.Please attempt the answers on the same paper and return it to the centre superintendent within the time allowed.

Q#1a)Mathch the entries of column Ato the correct entries of column B.(10)

Column A	Column B
Division ring is also known as	i)Linear algebra
$Z_6 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}\}$ has zero divisor.If n is composite,then it has	ii)±1
DimV(F)=$\{M_{2\times3}(F) set\ of\ all\ 2\times 3\ matrices\}$ then dim=	iii) feild
Hom(v,v) forms	iv)6
If A is orthogonal,then det A=	v)zero divisor
Null ring forms	vi)abelian
The centre of a group is	vii) 3
Klien 4 group is	viii)cyclic
Conjugate classes in S_3	ix)characterestic
(Q,+) is abelian but not	x)skew feild

B)Give short answers to the questions.(10)

i)Define centralizerof a subgroup.

ii)Define group homomorphism .

iii)Let $T: R^2 \to R^2$ be given by $T(x,y) = (-y,x)$,show that T is linear.

iv)Define invertiable element of a ring R.

v)Define Basis and dimensionof a vector space.

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GOVERNMENT COLLEGE UNIVERSITY,FAISALABAD

QUESTION PAPER EXTERNAL EXAMINATION

M.Sc Mathematics   Part 1st   1st Annual 2015

Course Code MTH-C12   Course Title: ALGEBRA   Pass Marks 40%

SUBJECTIVE PART	Time ALLOWED: 2:30 hrs	Marks 80

NOTE:Attempt any four questions by selecting two questions from each section.All questions carry equal marks.

Section 1

Q2.a) Prove that every subgroup of a cyclic group is cyclic.

b)Show that the set $\{1,-1,i,-i\}$ forms an abelian group under the multiplication of numbers.Is it a cyclic group?Find all of its generator. (10+10)

Q4.a) Prove that a group of order 66 is not simple.

b)Prove that if $\emptyset: G \to G'$ be a group homomorphisim from G into $G'$with $Ker\,\emptyset = K$.Then their exists an isomorphisim between $G/_K$ and $\emptyset(G)$. (10+10)

Q5.a) Define charactereristic subgroup.Prove that the centre Z(G) of G is a characteristic subgroup of G.

b)For any group G and $g \in G$.Show that g and g^{-1} has same order. . (10+10)

Section 2

Q6.a)prove that every finite integral domain forms a field. (10+10)

b)Define centre of a ring R and prove that the centre Z(R) of the ring R forms a subring of R.

Q7.a) If dim(V)=m and dim(W)=n,then prove that dim Hom(V,W)=mn. (10+10)

b)Consider the basis $\{V_1 = (2,1), V_2 = (3,1)\}$ of R^2.Find the dual basis $\{\emptyset_1, \emptyset_2\}$

Q8.a) Define change of basis. Consider the following two basis of R^2,$S = \{u_1, u_2\} = \{(1,2),(3,5)\}$, $S' = \{v_1, v_2\} = \{(1,-1),(1,-2)\}$.Find the change of basis matrix P for S to the new basis S'.

b)Find the change of basis matrix Q for the new basis S' back to the old basis S. (10+10)

Q9.a) For each of the following operators $T: R^2 \to R^3$, find all eigenvalues and a basis for each eigenspace.Also reduce the matrix of T to a diagonal matrix :$T(x,y,z) = (x+y+z, 2y+z, 2y+3z)$

b)Prove that any two eigenvectors corresponding to two distinct eigenvalues of an orthogonal matrix are orthogonal. (10+10)

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GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

QUESTION PAPER: EXTERNAL EXAMINATIONS

M Sc Mathematics　　Part 1st　　1st　　Annual 2015

Course Code: MATH-C13　　Course Title: Complex Analysis and Differential Geometry

OBJECTIVE PART　　Time Allowed: 30 Minutes　　Marks: 20

Note: *This question No.1 is compulsory and its all parts carry equal marks. Please attempt the answers on same paper and return it to center superintendent with in the time allowed.*

Q. No. 1

(a) Answer the following short questions

1) Find the upper bound for the absolute value of $\oint_c \frac{e^z}{z+1}dz$ where C is the circle $|z| = 4$

2) What is the equation of the envelope of the surface $x^2 + y^2 = 4a(f - a)$?

3) Consider the Serret-Frenet formulae, indicate what is the value of $\mathbf{b}''$?

4) If $-\overline{z} = z$ then explain the complex status of z.

5) Define envelope.

b) Match the column A with column B and select the relevant part from column B.

Colum A	Colum B
$\mathbf{r}'.\mathbf{r}''$	$\tau = 0$
Singular point	$\mathbf{r}_{11}$
$\mathbf{r}'.\mathbf{r}''\times\mathbf{r}''' = 0$	0
Polar form of complex number	Normal plane
$\frac{\partial^2 \mathbf{r}}{\partial u^2}$	Non-analytic
Reciprocal of Torsion	Centre of Torsion
Plane perpendicular to the tangent	$re^{i\theta}$
Cauchy's Integral formula	$f(z_0) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{z - z_0}$
$z\overline{z}'$	Weierstrass M-test
$\lvert f_n(x)\rvert \le M_n$	-1

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GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

QUESTION PAPER: EXTERNAL EXAMINATIONS

M Sc Mathematics	Part 1st	1st	Annual 2015
Course Code: MATH-C13	Course Title: Complex Analysis and Differential Geometry		
Time Allowed: 03:00 Hours	Maximum Marks: 100	Pass Marks: 40%	

SUBJECTIVE PART Time: 02:30 Hours Marks: 80

Note: *Attempt any four questions. All questions carry equal marks.*
Select two questions from each section.

Section I

Q1.

a) Find an upper bound for $\left|\frac{-1}{z^4-5z+1}\right|$, if $|z|=2$. [10]

b) Show that $v(x,y)=\frac{x}{x^2+y^2}$, is harmonic in a domain D not containing the origin. Find a function $f(z)=u(x,y)+iv(x,y)$ that is analytic in domain D. [10]

Q2.

a) Find image of a triangle with vertices 0, 1 and *i* under the mapping $f(z)=e^{i\frac{\pi}{4}z}$,represent the linear mapping with a sequence of plots. [10]

b) Find all solutions to the equation $\sin z=5$. [10]

Q3.

a) Consider a function f continuous in [0, 1] then prove that $F(z)=\int_0^1 f(z,t)\sin(zt)\,dt$, is analytic. [10]

b) Find an upper bound for the absolute value of $\oint_C \frac{e^z}{z+1}dz$ where C is a circle $|z|=4$. [10]

Q4.

a) Suppose that *f* is analytic in a simply connected domain *D* and *C* is any simple closed contour lying entirely within *D*. Then for any point z_0 within C, $f^n(z_0)=\frac{n!}{2\pi i}\oint_C \frac{f(z)}{(z-z_0)^{n+1}}dz$.

[10]

b) Find an upper bound for the absolute value of the given integral along the indicated contour. $\int_C \frac{1}{z^2-2i}dz$, where C is the right half of the circle $|z|=6$ from $z=-6i$ to $z=6i$. [10]

Section II

Q1.

a) If a fixed line makes an angle θ & φ with the unit vector $\bar{t}$ and the binomial $\bar{b}$, then find the relation between θ & φ [10]

b) Let "s" represents arc length of the curve $\bar{r}=\bar{r}(s)$ and "s_1" represents arc length of the locus of centre of curvature at a point $P(r)$ of the curve, then show that $\frac{ds_1}{ds}=\frac{1}{\kappa^2}\sqrt{\kappa'^2+\kappa^2\tau^2}$.

[10]

Q2.

a) Show the behavior of the curve if (i) $\kappa=0$ (ii) $\tau=0$ [10]

b) Show that; $\kappa=\frac{|\dot{\mathbf{r}}\times\ddot{\mathbf{r}}|}{|\dot{\mathbf{r}}|^3}$ & $\tau=\frac{\dot{\mathbf{r}}.\ddot{\mathbf{r}}\times\dddot{\mathbf{r}}}{\kappa^2|\dot{\mathbf{r}}|^6}$. [10]

Q3.

a) For the given surface. Find the fundamental magnitudes of 1st order. Also show that the parametric curves are orthogonal $\mathbf{r}=[u\cos v, u\sin v, f(u)]$. [10]

b) For orthogonal parametric curves find the D.E of the line on a surface cutting the curve u =constant. [10]

Q4.

a) Prove that a polar line is the axes of circle of curvature and the edge of regression of the polar developable is the locus of the centre of spherical curvature. [10]

b) Let κ_n be the normal curvature of a surface in any direction making angle α with a Principal direction then $\kappa_n=\kappa_a\cos^2\alpha+\kappa_b\sin^2\alpha$ [10]

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GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

QUESTION PAPER: EXTERNAL EXAMINATIONS

M. Sc. Mathematics — Part 1st — 1st Annual 2015
Course Code: Math-C14 — Course Title: Mechanics

OBJECTIVE PART — Time Allowed: 30 Minutes — Marks: 20

Note: *This question No.1 is compulsory and its all parts carry equal marks. Please attempt the answers on same paper and return it to center superintendent with in the time allowed.*

Q. No. 1 (a) Answer the following short questions (5x2)

(i) Exprss the plane $y = x$ in spherical coordinate system
(ii) If ϕ is a differentiable scalar function then provethat $curl\ grad\ \phi = 0$.
(iii) Fined function $f(r)$ such that $\nabla^2 f(r) = 0$.
(iv) Differentiate between body and space cone.
(v) Show that first partial derivative $\partial\phi/\partial x$ is the directional derivative in the direction of x-axis.

Q. No. 1 (b) Match the column A with the column B and select the relevant part from column B. (10x1)

	Column A	Column B	Column C
i	$\mathbf{A}\times\mathbf{B}$	0	
ii	$\nabla\times\mathbf{B}$	Gauss Divergence Theorem	
iii	$\frac{\partial\phi}{\partial x}$	0	
vi	$\nabla.\nabla\times\mathbf{A}$	3	
v.	$\int_c(\mathbf{A}.\hat{n}ds) = \iiint_R \nabla.\mathbf{A}dv$	$C_i = \varepsilon_{ijk}A_jB_k$	
vi	$\int_c(Mdx + Ndy) = \iint_R\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$	$\frac{1}{r^2}$	
vii	$\nabla.\mathbf{r}$	Directional derivative along x-axis	
viii	$\nabla^2\left(\frac{1}{r}\right)$	Greeen's Theorem	
xi	$\nabla^2(\ln(r))$	**B** is conservative force field	
x	$\nabla\times\mathbf{r}$	$C_i = \varepsilon_{ijk}A_jB_k$	

GOVERNMENT COLLEGE UNIVERSITY, FAISALABA

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QUESTION PAPER: EXTERNAL EXAMINATIONS

M. Sc. Mathematics   Part 1st   1st Annual 2015

Course Code: Math-504 /C14   Course Title: Mechanics

Time Allowed: 03:00 Hours   Maximum Marks: 100   Pass Marks: 40%

SUBJECTIVE PART   Time: 02:30 Hours   Marks: 80

Note: *Attempt any four questions by selecting two questions from each section. All questions carry equal marks.*

Section I

Q. No. 2 (a) For any arbitrary constant vector $\mathbf{A}$ prove that $\nabla\left(\frac{\mathbf{A.r}}{r^3}\right)+\nabla\times\left(\frac{\mathbf{A}\times\mathbf{r}}{r^3}\right)=0$

(b) Prove that $\mathbf{a}.\left(\nabla(\mathbf{V}.\mathbf{a})-\nabla\times(\mathbf{V}\times\mathbf{a})\right)=\nabla.\mathbf{V}$, where $\mathbf{a}$ is a constant unit vector.

Q. No. 3 (a)) Use green theorem to in the plane to evaluate $\oint_C\left(x^2-2xy\right)dx+(x^2y+3)dy$, by around the boundary of the region defined by $y^2=8x,\ x=2$.

(b) Prove that the cylindrical system is orthogonal curvilinear coordinate

Q. No. 4 (a) prove by using tensor methods $\nabla.\left(\nabla\times\mathbf{A}\right)=0$

(b Prove that product $\mathbf{A}_i\mathbf{B}_j\mathbf{C}_k$ is a tensor of rank three where $\mathbf{A}_i$, $\mathbf{B}_j$, $\mathbf{C}_k$ are tensors of rank and $i,j,k=1,2,3$.

Q. No. 5 (a) Prove that if $\mathbf{A}_{jk}$ and $\mathbf{B}_{jk}$ are tensors of rank 2 then addition and subtraction of these tensors are tensors of rank 2.

(b)) If A_{ij} is a second symmetric tensor, show that $\varepsilon_{ijk}A_{ij}=0$ is a third order tensor for all values of k, also prove the converse that if A_{ij} is a second order tensor and $\varepsilon_{ijk}A_{ij}=0$ is a third order tensor for all values of k, then show that A_{ij} is symmetric.

Section II

Q. No. 6 (a) A rigid body consists of three particles of mases 2,1,4 kgs located at the points (1,-1,1), (2,0,2), (-1,1,0) respectively find the angular momentum of the body if it is rotated about the origin with angular velocity $\omega=3\hat{i}-2\hat{j}+4\hat{k}$.

(b) A rigid body S has spin $\boldsymbol{\omega}$ and a particle Q of S has velocity $\mathbf{V}$. Show that every particle P of S with velocity vector parallel to $\boldsymbol{\omega}$ lies on the line $\mathbf{QP}=\frac{\boldsymbol{\omega}\times\mathbf{V}}{\boldsymbol{\omega}^2}+\mu\boldsymbol{\omega}$, where μ is an arbitrary parameter.

Q. No. 7 (a) A hollow sphere of metal of uniform density has internal and external radii a, b respectively. Find its moments of inertia about any tangent line.

(b) Sate and prove parallel axes theorem.

Q. No.8 (a) Find the principal moments and principal axes of inertia for a uniform rectangular plate of sides a, b at its center.

(b) Show that a uniform solid cuboid of Mass M is equi-momental with

i) masses (1/24)M at midpoints of its edges and (1/2)M at its center.

ii) masses (1/24)M at its corners and (2/3)M at its center.

Q. No. 9 (a) Show that the moment of inertia of a uniform solid right circular cone of mass M, height h and semi vertical angle α about a diameter of its base is $\frac{Mh^2(3\tan^2\alpha+2)}{+20}$.

(b) Derive Euler's equation for rigid body motion in a force field. Use these to obtain a complete solution of the problem of free rotation of symmetrical rigid body.

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GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

QUESTION PAPER: EXTERNAL EXAMINATIONS

M Sc Mathematics
Course Code: MATH-C15

Part 1st
Course Title: Topology and Functional Analysis

1st / 2nd Annual 2015

OBJECTIVE PART Time Allowed: 30 Minutes Marks: 20

Note: *This question No.1 is compulsory and its all parts carry equal marks. Please attempt the answers on same paper and return it to center superintendent with in the time allowed.*

Q.No.1 (a) **Encircle the correct answer.(Topology and metric on $\mathbb{R}, \mathbb{R}^2$ are standard, mentioned otherwise)**

(1): Let $\mathbb{N} = \{1, 2, 3, ...\} \subset \mathbb{R}$ then subspace topology on $\mathbb{N}$ is
(a) not discrete (b) discrete (c) co finite (d) indiscrete

(2): X is disconnected if there is $A \subsetneq X$ which is
(a) closed (b) open (c) both open and closed (d) none of these

(3): Open ball of radius 2 and center 1 in $\mathbb{R}$, is
(a) $(1, 2)$ **(b)** $(-1, 3)$ **(c)** $[-1, 3]$ **(d) none of these**

(4) Let $(X, <.,.>)$ be a complex inner product space. Then $< ix, (i-1)y >$ can be written as:
(a) $(-i+1) < x, y >$ **(b)** $i < x, y >$ **(c)**$(i-1) < x, y >$ **(d)** $(1+i) < x, y >$

(5)Let $x, y \in \mathbf{R}^2$. Then $< x, y >= 0$ if
(a)$x = (3, 1), y = (1, 3))$ **(b)** $x = (1, 0), y = (0, 1)$ **(c)** $x = (3, -2), y = (2, -1)$ **(d)** $x = (0, -1), y = (2, 1)$

(b) **Answer the following short questions**

(i) **Define hausdorff topological space.**

(ii) **What is an open cover of a topological space X.**

(iii) **Define inner product Space.**

(iv)**What is continous linear operator.**

(v) **Define continous mapping on metric space X to metric space Y.**

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GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

QUESTION PAPER: EXTERNAL EXAMINATIONS

M Sc Mathematics | Part 1st | 1st | Annual 2015

Course Code: MATH-C15 | Course Title: Topology and Functional Analysis

Time Allowed: 03:00 Hours | Maximum Marks: 100 | Pass Marks: 40%

SUBJECTIVE PART | Time: 02:30 Hours | Marks: 80

Note: Attempt any four question by selecting two questions from each section. All questions carry equal marks.

Section 1

Q.No.2 (a) Let A be a subset of a topological space then $\delta(A) \subset (A)$ if and only if A is closed .
(b) For sets A and B in a topological space satisfies

$$int(A) \cap int(B) = int(A \cap B)$$

Q.No.3 (a) Prove that in a topological space X, if U is open and C is closed then $U - C$ is open and $C - U$ is closed.
(b) Let X be a Hausdorff topological space and A be compact in X then A is closed in X.

Q.No.4 (a) Prove that if A be a subset of a topological space X then A is open iff $A = int(A)$.
(b) Let $f : X \to Y$ be a continous map from topological space X to Y then if A is connected in X implies $f(A)$ is connected in Y.

Q.No.5 (a) Prove that $\mathbb{R}$ with standard topology is homeomorphic to $(0, 1)$.
(b) A function $f : X \to Y$ from topological space X to topological space Y is continous if and only if $f^{-1}(C)$ is closed in X for every closed set C of Y.

Section 2

Q.No.6 (a) Show that $d(x, y) = \sqrt{|x - y|}$ defines metric on $\mathbb{R}$
(b) Prove that in a metric space (X, d), every convergent sequence is Cauchy sequence. What about its converse.
Q.No.7 (a) If a normed space is finite dimensional then every linear operator is bounded.
(b) Let $T : X \to Y$ be a linear operator on normed spaces X and Y, the T is cotinous if and only if T is bounded.
Q.No.8 (a) A subspace Y of a Banach space X is complete iff Y is closed in X.
(b) Give an example of linear operator which is not contious.
Q.no 9 Let H_1, H_2 be two Hilbert Spaces and $T_1 : H_1 \to H_2$, $T_1 : H_1 \to H_2$ be bounded linear operators, α be any scalar then
(a)$(T_1T_2)^* = T_2^*T_1^*$
(b) $(\alpha T_1)* = \overline{\alpha}T_1^*$.

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GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

QUESTION PAPER: EXTERNAL EXAMINATIONS

M Sc Mathematics | Part 2nd | 1st | Annual 2015
Course Code: MTH-D11 | Course Title: Fluid Mechanics

OBJECTIVE PART

Time Allowed: 30 Minutes | Marks: 20

Note: This question No.1 is compulsory and its all parts carry equal marks. Please attempt the answers on same paper and return it to center superintendent with in the time allowed.

Q. No. 1 (a) Answer the following short questions

(i) Define irrotational flow.

(ii) Write the equation of vortex line.

(iii) Define stream line and path line.

(iv) Define equation of impulsive motion.

(v) Write the expression for material derivative, local derivative and convective derivative.

Q. No. 1 (b)

	Column A		Column B
i.	Equation of continuity	i.	$\frac{1}{2}q^2+\Omega+\int\frac{dp}{\rho}=c$
ii.	Bernoulli equation	ii.	$\nabla^2\phi=0$
iii.	Laplace equation	iii.	$\frac{\partial}{\partial t}$
iv.	Local acceleration	iv.	$\frac{D\vec{q}}{Dt}=\vec{F}-\frac{1}{\rho}\nabla p$
v.	Euler's equation	v.	∇^2
vi.	Laplacian	vi.	$\vec{q_1}-\vec{q_2}=\vec{F}-\frac{1}{\rho}\nabla p$
vii.	Impulsive motion	vii.	$\nabla\cdot\vec{q}=0$
viii.	Naviour Stockes equation	viii.	$\nabla\times V\neq 0$
ix.	Rotational fluid motion	ix.	None of these
x.	Newtonian law	x.	$\rho g-\nabla\rho+\nabla\cdot\tau=\rho\frac{DV}{Dt}$

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GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

QUESTION PAPER: EXTERNAL EXAMINATIONS

M Sc Mathematics | Part 2nd | 1st Annual 2015
Course Code: MTH-D11 | Course Title: Fluid Mechanics

Time Allowed: 03:00 Hours | Maximum Marks: 100 | Pass Marks: 40%

SUBJECTIVE PART | Time: 02:30 Hours | Marks: 80

Note: Attempt any four questions. All questions carry equal marks. (Two questions from each section)

SECTION- I

Q. No.2 (a) Derive the equation of continuity in vector form.

(b) Define material derivative local derivative and convective derivative also write the material derivative in cylindrical coordinates.

Q. No.3 (a) Define the relation between stream function and potential function.

(b) State and prove Newton's law of viscosity.

Q. No.4 (a) Define complex velocity potential, source and sink give one example of each.

(b) Derive relation between velocity potential and stream function in Cartesian form.

Q. No.5 (a) Derive Euler equation in Cartesian form.

(b) Define impulsive motion also derive the equation of impulsive motion.

SECTION- II

Q. No.6 (a) Find the condition for a surface to be boundary surface when $\frac{\partial F}{\partial t} = 0$

(b) Discus the flow of viscous fluid between two parallel plates when both plates are at rest, also find the expression for velocity field.

Q. No.7 (a) Derive the Navior stockes equations in Cartesian coordinates.

(b) Write the limitation of Navior stockes equations also write the Navior stockes equations in cylindrical form.

Q. No.8 (a) Define vortex line and derive the differential equation of vortex line.

(b) Find the condition for $F(\vec{r},t)$ to be a boundary surface.

Q. No.9 (a) Define a doublet also find the expression for complex potential due to a doublet.

(b)what arrangement of souses and sink will give rise to the $\log\left(Z - \frac{a^2}{Z}\right) = W$ draw rough sketch of stream line.

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GOVERNMENT COLLEGE UNIVERSIT, FAISALABAD

M Sc. Mathematics Part 2nd 1st Annual 2015

Course Code: MTH-602/D12 Course Title : Mathematical Methods and Partial Diff. Equs.

OBJECTIVE PART Time Allowed: 30 Minutes Marks: 20

Note: This question No. 1 is compulsory and all its parts carry equal marks. Please attempt the answers on the same paper and return it to the center superintendent within the time allowed.

Q No. 1. (a). Give the short answers. (2x5=10)

(i) Define the basis of a differential equation.
(ii) What is the condition for dependent and independent solution of a differential equation.
(iii) Describe method of separation of variables for PDEs.
(iv) Define Laplace transform.
(v) Define a singular integral equation.

(b). Match the column A with the column B and select the relevant part from column B. (1x10=10)

Column A	Column
i. $\frac{d^2y}{dx^2}+y\ln y=0$	a. $n!$
ii. Eigenfunctions & $\frac{d^2y}{dx^2}+\lambda y=0, y(0)=y(L)=0$	b. unit step function
iii. $J_{-n}(x)$	c. $F(s)G(s)$
iv. Green's functions $g(x, s)$	d. $\frac{e^{-k^2/4}}{\sqrt{2}}$
v. $J_{-1/2}(x)$	e. $\lvert f(t)\rvert < Me^{ct}$ for t>T
vi. $\int_0^\infty e^{-x}x^{n-1}dx$	f. nonlinear and homogeneous equation
vii. $u_a(t)$	g. $\sqrt{\frac{2}{\pi x}}\cos x$
viii. $L[f*g]$	h. continuous at x=s
ix. Fourier transform of e^{-x^2}	i. $\sin(m\pi x/L)$
x. function of exponential order	j. $(-1)^n J_n(x)$

GOVERNMENT COLLEGE UNIVERSIT, FAISALABAD

M Sc. Mathematics Part 2nd 1st Annual 2015

Course Code: MTH-602/D12 . Course Title : Mathematical Methods and Partial Diff. Equs.

SUBJECTIVE PART Time Allowed: 2-1/2 hours Marks: 80

Note: *Attempt any four questions by selecting two questions from each section. All questions carry equal marks.*

SECTION 1

Q 2 (a). Prove that the eigenvalues equations related to the SL system: $\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right]+q(x)y+\lambda r(x)y=0$ with BCs: $a_{11}y(a)+a_{12}y'(a)=0$, $a_{21}y(b)+a_{22}y'(b)=0$., are real.

(b). Find the eigenvalues and eigen-functions associated with the following boundary problem: $y''+\lambda y=0$ under boundary conditions: $y'(0)=y'(L)=0$. Find its Fourier Legendre's series over $[0,L]$

Q 3 (a). Prove Rodrigue formula : $P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$ for Legendre polynomials $P_n(x)$ of degree n.

(b). Show that $J_{-n}(x)=(-1)^n J_n(x)$ where n is a positive integer. Evaluate $J_{3/2}(x)$ in terms of sine and cosine functions

Q 4. (a) Using the Green function, determine the solution of the boundary value problem:

$$c^2y''=e^x \text{ subject to the boundary conditions: } y(0)=y(1)=0$$

(b). Find the Green function associated with the following boundary value problem:

$$y''-y=f(x),\ y(0)=1,\ y(\pi/2)=2$$

5. (a). Define degenerate kernel. Determine the resolvent kernel for the following integral equation:

$$g(x)=f(x)+\lambda\int_{-1}^{1}(xt+x^2t^2)g(t)dt$$

(b). Use the successive approximation method to solve the integral equation:

$$g(x)=\sin x-\frac{1}{4}x+\frac{\lambda}{4}\int_{0}^{\pi/2}xtg(t)dt$$

P.T.O

SECTION 2

Q 6. (a). Solve: $\varphi_{xx} = \varphi_{tt}$ subject to BCs: $\varphi(0,t) = 0, \varphi(L,t) = p$. ICs: $\varphi(x,0) = f(x)$.

(b). Solve the PDE: $\phi_{xxxx} + \phi_{yyyy} = \phi_{tt}$.

Q 7. (a). Show that the equation : $e^{x}\phi_{xx} - e^{y}\phi_{yy} = 0$ is always hyperbolic and convert it into canonical form.

(b). Write the Laplace equation into polar coordinates and solve it.

Q 8. (a). Find the Fourier transform of $f(x) = Ne^{-ax^2}$

(b). Use the Fourier transform to solve the equation: $y'' - y = g(x)$

9. (a). Solve the differential equation: $\dfrac{\partial u(x,t)}{\partial t} = c^2 \dfrac{\partial^2 u(x,t)}{\partial x^2}$

$u(x,0) = T_0$, $u(0,t) = 0$, $u(x,t) \to 0$ as $x \to \infty$.

(b). Use the Laplace Transform to solve the IVP:

$y'' - ay = f(t)$ with ICs: $y(0) = p_1$, $y'(0) = p_2$.

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GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

QUESTION PAPER: EXTERNAL EXAMINATIONS

M Sc Mathematics — Part 2nd — 1st Annual 2015

Course Code: MATH-603/D5 — Course Title: Mathematical Statistics

OBJECTIVE PART — Time Allowed: 30 Minutes — Marks: 20

Note: *This question No.1 is compulsory and its all parts carry equal marks. Please attempt the answers on same paper and return it to center superintendent with in the time allowed.*

Q. No. 1

(a) Answer the short questions **($2\times5=10$)**

(i) A student selected from a class will be either a boy or girl. If the probability that a boy will be selected is 0.3, what is the probability that a girl will be selected ?

(ii) If A and B independent events then show that

$$P(A\cup B)^c = P(A^c)\,P(B^c).$$

(iii) Check whether the given function

$$f(x)=\frac{1}{5} \quad \text{for } x = 1, 2, 3, 4, 5$$

Can serve as probability distribution function for a random variable 'X'?

(iv) What is the difference between parameter and statistics ?

(v) Define the sufficiency of an estimator .

(b) Match the column A with column B and select the relevant part from column B. Write the correct answer in column C, parallel to each question. **($1\times10=10$)**

A	B	C
(i) Discrete random variable 'X'	Quantity of milk	
(ii) To check the hypothesis about σ^2	χ^2 - distribution	
	The Birthday problem	
(iii) Sampling with replacement	F- distribution	
(iv) Unbiased estimator		
(v) X and Y are independent random variables	$f(x_1,\ldots,x_n,\theta)=g(\hat{\theta},\theta)\,h(x_1,\ldots,x_n)$	
(vi) $\hat{\theta}$ is sufficient estimator of θ	E(XY)=0	
(vii) Sampling without replacement	Alternative hypothesis	
(viii) Hypothesis about ratio of two variances	$\frac{n!}{(n-k)!}$	
(ix) Continuous random variable 'X'	$E(X)=\sum_{x\in X} x\,f(x)$	
(x) H: $\theta<\theta_0$	$E(\hat{\theta})=\theta$	

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GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

QUESTION PAPER: EXTERNAL EXAMINATIONS

M Sc Mathematics   Part 2nd   1st   Annual 2015
Course Code: MATH-603 /DST   Course Title: Mathematical Statistics

Time Allowed: 03:00 Hours   Maximum Marks: 100   Pass Marks: 40%

SUBJECTIVE PART   Time: 02:30 Hours   Marks: 80

Note: *Attempt any four questions. All questions carry equal marks.*

Imp. Note— **Select two questions from each section:**

Section-1

Q-No.2: (a) Prove that for any two events A and B , the probibilty that exactly one of the two events will occur is given by the expression

$$P(A) + P(B) - 2\,P(A \cap B)$$

(b) Let $A_1, A_2, A_3 \ldots$ be an infinite sequence of events such that $A_1 \supset A_2 \supset A_3 \supset \ldots$.

Prove that $P(\bigcap_{i=1}^{\infty} A_i) = \lim_{n \to \infty} P(A_n)$

Q-No.3: (a) A box contains 24 light bulbs, of which 2 are defective. If a person select 10 bulbs at random without replacement, what is the probability that both defective bulbs will be selected?

(b) State and prove the Baye's theorem?

Q-No.4: Suppose that the probability density function of a random variable 'X' is as follow:

$$f(x) = \begin{cases} ce^{-2x} & for \quad x > 0 \\ 0 & otherwise \end{cases}$$

(a) Find the value of 'c'?
(b)Find the value of $P(1 < X < 2)$?

Q-No.5: Define the Normal random variable 'Z' ? Find the expectation and variance of 'Z'?

Section-2

Q-No.6: Suppose that $X_1, X_2, ..., X_n$ form a random sample from an Exponential distribution for which the value of parameter β is unknown. Determine the M.L.E of the mean of the distribution ?

Q-No.7: Find the mode of the χ^2 – distribution with 'n' degree of freedom (n=1 , 2 , …).

Q-No.8: The following table gives the census data of orchards. Test the hypothesis that the two variables of classification are independent

Classes	Shaded	Unshaded	Total
Highly Yielders	350	205	555
Low Yielders	250	195	445
Total	600	400	1000

Q-No.9: Ten individuals are chosen at random from a normal population and the heights are found to be in inches 63 , 63 , 66 , 67 , 68 , 69 , 70 , 70 , 71 and 71. In the light of these data, discuss the suggestion that mean height in the population is 66 inches?

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GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

MSc Mathematics Part 2^{nd} 1st Annual 2015

Course Code: Math-604/D52 Course Title: Numerical analysis

Objective Part Time: 30 Minutes Marks: 20

Question # 1 a) Answer the following short questions

(i) What is the difference between regula falsi method and secant method

(ii) Define order of convergence of the iterative methods

(iii) State Weiestrass Approximation Theorem.

(iv) Why the least square method is called least square.

(v) State the sufficient condition for fixed point solution.

Question # 1 b) Match the column A and Column B

Column A	Column B
Bi sec tion Method	Linear
Error term in Simpsons $\frac{1}{3}$ rule	Direct
Accuracy of Trapezoidal rule	Quadratic
Interpolating Polynomials	Linear
$n+1$ data points	$-\frac{1}{90}h^5 f^{(iv)}(\xi)$
Gauss Elimination Method	Initial Point
Re gula falsi method	$-\frac{3}{80}h^5 f^{(iv)}(\xi)$
Error term in Simpsons $\frac{3}{8}$ rule	Unique
Secant Method	Polynomial of degree n
Newton Method	Converges

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GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

MSc Mathematics | Part 2^{nd} | 1st Annual 2015

Course Code: Math-604/D52 | Course Title: Numerical analysis

Time Allowed: 03:00 Hours | Maximam Marks: 100 | Pass Marks: 40%

Subjective Part | Time: 2:30 Hours | Marks: 80

1. SECTION1

Question #2 a) Prove that the error bound in the interval $[a, b]$ for bisection method is $|\alpha - c_n| \leq \frac{1}{2^n}(b-a)$, α is the exact solution and c_n is the approximated soltion.

b) Prove that the bisection method converges to the exact solution and

$$n \geq \frac{\log\left(\frac{b-a}{\epsilon}\right)}{\log 2},$$

where n is the number of iterations required for the error bound ϵ.

Question #3 a) Let $f(x) = 1 - x - \sin x = 0$. Find the interval $[a, b]$ containing α and for which the bisection method converges to α. Then find the number of iterations required to estimate α with in accuracy of 5×10^{-8}.

b) .Find the zeros and of multiplicity of the function

$$f(x) = \cos x + e^x - \frac{x^4}{12} - \frac{x^3}{6} - x - 2.$$

Question #4 a) Let $g \in C[a, b]$ such that $g(x) \in [a, b]$, $\forall\, x \in [a, b]$. Suppose that g' exists on (a, b) and there exists a constant $0 < k < 1$ such that

$$|g'(x)| \leq k, \ \forall\, x \in (a, b).$$

Then $x = g(x)$ has unique solution.

b) Then prove that for any number p_0 in $[a, b]$ the approximate solution p_{n+1} obtained by the iterative scheme

$$p_{n+1} = g(p_n), \quad n = 0, 1, 2, \ldots$$

converges to the unique fixed point p in $[a, b]$. Also show that

$$|p_n - p| \leq \frac{k^n}{1-k}|p_1 - p_0|, \ \forall\, n \geq 1.$$

Question #5 a) Find the approximated solution of the linear system by using Gauss Seidal method

$$\begin{aligned} 10x_1 - x_2 + 2x_3 &= 6, \\ -x_1 + 11x_2 - x_3 + 3x_4 &= 25, \\ 2x_1 - x_2 + 10x_3 - x_4 &= -11, \\ 3x_2 - x_3 + 8x_4 &= 15. \end{aligned}$$

b) Construct $P_2(x)$ for the data points $(0, -1)$, $(1, -1)$ and $(2, 7)$ by using Langrange polynomial.

P.T.O

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GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

QUESTION PAPER: EXTERNAL EXAMINATIONS

1st Annual 2015

M Sc Mathematics Part 2nd

Course Code: MATH-605/D53 Course Title: Operational Research & C++

OBJECTIVE PART Time Allowed: 30 Minutes Marks: 20

Note: *This question No.1 is compulsory and its all parts carry equal marks. Please attempt the answers on same paper and return it to center superintendent with in the time allowed.*

Q. No.1

(a)

(i) Discuss the various general classes of problem that LP can help in solving them.

(ii) Differentiate between basic and non-basic variables determined for the initial simplex tableau?

(iii) Write the dual of the following LP problems:

$$\max Z = 5x_1 + 12x_2 + 4x_3$$
$$x_1 + 2x_2 + x_3 \le 16$$
$$2x_1 - x_2 + 3x_3 = 8$$
$$x_1, x_2, x_3 >= 0.$$

(iv) What is an assignment problem? Describe the relation between the transportation and assignment problems.

(v) Distinguish between the standard simplex method and dual simplex method.

(b) Match the column A with column B and select the relevant part from column B 1×10

Column A	Column B
Represent the physical limitation of the Problem	Multiple optimal solution
The simplex method begin with a	For remainder
Transportation problem deals with	Global variable
Assignment problem is actually	Real type data
The branch and bound method	Tree search
Feasible region	Physical movement of goods
An LP problem may have	Basic feasible solution
The float represents	Constraints
% means	General transportation problem
Variables declared outside the main function	Admissible region

Roll No..........

GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

QUESTION PAPER: EXTERNAL EXAMINATIONS
1st Annual 2015

M Sc Mathematics
Course Code: MATH-605/D53
Part 2nd
Course Title: Operational Research & C++

Time Allowed: 03:00 Hours | Maximum Marks: 100 | Pass Marks: 40%

SUBJECTIVE PART | Time: 02:30 Hours | Marks: 80

Note: *Attempt any four questions. All questions carry equal marks.* Selecting two questions from each section.

Section-I

Question # 2: Solve the following LP problem using the simplex method:

$$\max Z = 5x_1 + 7x_2$$
$$x_1 \le 10$$
$$x_1 + x_2 = 12$$
$$x_1 - 2x_2 >= 3$$
$$x_1, x_2 >= 0.$$

Question # 3: Solve the following ILP problem using the cutting-plane method:

$$\max Z = 4x_1 + 5x_2$$
$$3x_1 + 5x_2 \le 40$$
$$x_1 + x_2 \le 9$$
$$x_1, x_2 >= 0; x_1 \text{ integer}.$$

Question # 4: PIA has 26 aircrafts with 20 passenger seats and 13 aircrafts with 30 passenger seats to be used for Gilgit and Azad Kashmir routes. Each 20 seater aircraft needs one pilot and two cabin crews while a 30 seater aircraft needs one pilot and five cabin crews. PIA intends to carry at least 420 passengers on one of the routes and has at the most 60 cabin crews available for duty. What is the minimum number of pilot PIA has to employ?

Question # 5: A company manufactures iron windows in three factories and ship them to four distribution points. The per window shipping costs are as follows:

Distribution Points

From \ To:	A	B	C	D	Capacity (units)
Factory 1	10	8	12	14	300
Factory 2	6	7	10	11	150
Factory 3	4	9	7	5	250
Demand (units)	350	125	100	125	

Find the initial feasible shipping allocation using North-west corner method.

Section-II

Question # 6: (a) Write a program to print a message on screen using "cout" object.
(b) Write a program to assign a value 515 to integer type variables x, y, a, b and c. Also calculate the sum of the variables and print the result on the screen.

Question # 7: Write a program in C++ to pass an integer value to a function so that the function returns the integer with its digits reversed, e.g. to return 4567 as 7654.

Question # 8: Write a program in C++ to exchange the values of two variables by using pointers.

Question # 9: What will be the value of c after executing the following statements if a=29 and b=6. (i) c=a&b (ii) c=a>>b

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GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

QUESTION PAPER: EXTERNAL EXAMINATIONS

M Sc Mathematics Part 2nd 1st Annual 2015

Course Code: MATH-D55 Course Title: Theory of Modules and Theory of Optimization

OBJECTIVE PART Time Allowed: 30 Minutes Marks: 20

Note: *This question No.1 is compulsory and its all parts carry equal marks. Please attempt the answers on same paper and return it to center superintendent with in the time allowed.*

Q. No. 1

(a) Short answer question (5x2)

(i) Explain is quotient module.

(ii) Define the basis of the free module with examples.

(iii) Explain the Linear programming with real life examples.

(iv) Define Euclidean domain.

(v) Elaborate convex set and convex cones.

(b) Match the column.

1 slack variables	minimum
2 constrained optimisation	local minima
3 Any R-module M has the ---- M and {0}.	empty set
4 If Constraints are inconsistent then solution	bounded
5 An R-module M is called --- if M can be generated by one element	sub modules
6 A is a cyclic z-module if and only if A is a ---- group.	cyclic
7 Zero module is freely generated by the ----	Lagrange multiplier
8 if $f^{(iv)} > 0$	does not exist
9 If feasible region is bounded in an optimisation problem then solution is also ---	Represent the quantity of unused resources.
10 region of solution in optimisation is always ---	non - negative

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GOVERNMENT COLLEGE UNIVERSITY, FAISALABAD

QUESTION PAPER: EXTERNAL EXAMINATIONS

M Sc Mathematics | Part 2^{nd} | 1st | Annual 2015

Course Code: MATH-D55 | Course Title: Theory of Modules and Theory of Optimization

Time Allowed: 03:00 Hours | Maximum Marks: 100 | Pass Marks: 40%

SUBJECTIVE PART | Time: 02:30 Hours | Marks: 80

Note: *Attempt ~~any~~ four questions. All questions carry equal marks.* Attempt two Question from each Section.

Section: A

Q.1 Prove if L_1 and L_2 are sub modules of an R-module M, then $L_1 + L_2$ is also a sub modules of M.

Q.2 prove that

(i) ker Ø is an R-submodule of M.

(ii) ker Ø is an R-submodule of N.

Q.3 Find all the units of the integral domain of Gaussian integers.

Q.4 let θ be a module homomorphism show that θ is an isomorphism if and only if Ker (θ) = {0}

Section: B

Q.1 the cost of construction of a house is mainly according to its covered area wall space is wasted area. For optimal space utilization what should be design of the outside walls for a house (with straight edges) (at right angles).

Q.2 Find the extremal vales (if only) of

$f(x,y) = -x- y+ e^{xy}$

Q.3 Optimize $f(x, y) = x^2 + y^2$ subject to the constraints $Ø(x, y) = x + y - 1 = 0$

Q.4 solve with Kuhn-Jucker with diagram.

Max, $f(x, y) = x + 2y$ subject to $3x^2 + y^2 < 1$

$X - 8 \le -1 \quad (x, y \ge 0)$